

Preservation of the geometric quantum discord in noisy environments

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Geometric description of quantum correlations are favored for their distinct physical significance. Geometric discord based on the trace distance and the Bures distance are shown to be well-defined quantum correlation measures. Here, we examine their particular dynamical behaviors under independent as well as common structured reservoirs, and reveal their robustness against decoherence. We showed that the two well-defined geometric discord may be preserved well, or even be improved and generated by the noisy process of the common reservoir. Moreover, we also provided a strategy for long-time preservation of these two geometric discord in independent reservoirs.

PACS numbers: 03.65.Ud, 03.65.Ta, 03.67.Mn

Key Words: Quantum discord; Trace distance; Bures distance

I. INTRODUCTION

The existence of quantum correlations in a system is one of the most remarkable features of quantum theory which differentiates the quantum world from that of the classical one, and quantifying and understanding quantum correlations remains the subject of active research since the early days of quantum mechanics [1]. In the past two decades, a broad survey of different aspects of quantum correlations, such as the Bell-type correlations [2], the capacity for teleportation [3], and a plethora of measures for quantum entanglement [1], were performed. Particularly, since the pioneering work of Ollivier and Zurek [4], and that of Henderson and Vedral [5], the concept of quantum discord (QD) as a more general quantum correlation measure than that of entanglement, prompted a huge surge of people's research interest from different perspectives, see Refs. [6, 7] for a comprehensive review.

Originally, the QD was defined through the discrepancy between two expressions of the mutual information that are classically identical and quantum-mechanically inequivalent [4]. This is indeed an entropic measure of quantum correlation, and was favored for its operational interpretations [8–11] and potential applications in various quantum tasks [12–14]. But its evaluation is very hard due to the optimization procedure involved, and the closed expressions are known only for certain special (such as the Bell-diagonal [15]) states. Particularly, it has been shown that analytical evaluation of QD for general states is impossible [16]. Therefore, other measures of quantum correlations which are easy to calculate are needed. In this respect, Luo presented the concept of measurement-induced disturbance [17], where the measurement is induced by the spectral resolutions of the reduced states of a system. As such, it evades the procedure of optimization which is usually intractable.

Another routine for characterizing quantum correlations is via the geometric approach based on different distance measures. The seminal work along this line was that accomplished

by Dakić *et al.* [18]. They proposed to use the square of the minimal Hilbert-Schmidt distance as a basis for defining QD, and subsequently, Luo and Fu presented a variational and equivalent definition for it based on von Neumann measurements, and derived a tight lower bound for general bipartite states [19]. The figure of merit for this geometric measure of QD lies in its analytical evaluation for general two-qubit states [18] and certain bipartite states with high symmetry [19, 20]. It also plays a crucial role in specific quantum protocols, such as remote state preparation [21]. But this measure of geometric discord may be increased by trivial local operations on the unmeasured subsystem [22], and thus was not a good measure of quantum correlations [5]. To avoid this shortcoming, geometric discord based on other distance measures were proposed, e.g., the modified version of the geometric discord defined by making use of the square root of the density operator [23]. Here, we will consider the geometric discord defined by employing the trace distance [24] and the Bures distance [25]. This way of characterizing quantum correlation has previously been suggested by Luo and Fu in their pursuing analytical solutions for the geometric discord [19], and has been exploited explicitly very recently [24–26]. They can circumvent the problem occurs for the geometric discord defined in Ref. [18], and therefore can be regarded as well-defined measures of quantum correlations.

From a practical point of view, one may wonder its robustness against decoherence after the introduction of a new well-defined quantum correlation measure. Due to the advantage of the distance measures adopted for defining quantum correlations, it is expected that the aforementioned two geometric discord will exhibit different behaviors under decoherence [26], and a comparative study of this issue may provide us with information that is essential to various quantum protocols, particularly those based only on them.

In this paper, we take an investigation of the above problem. To be explicitly, we consider robustness of the foregoing two well-defined geometric discord for a central two-qubit system coupled to noisy environments. We will compare their particular dynamical behaviors, and try to provide effective methods for fighting against the deterioration of them, as this is of special importance to various quantum protocols.

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II. WELL-DEFINED MEASURES OF GEOMETRIC DISCORD

To begin with, we first briefly review the definitions as well as the general formalism for the trace distance and the Bures distance geometric discord. For a bipartite system AB described by the density operator ρ , the trace distance discord is defined as the minimal trace distance between ρ and all of the classical-quantum states ρ_{CQ} [24], namely,

$$D_T(\rho) = \min_{\chi \in \rho_{CQ}} \|\rho - \chi\|_1, \quad (1)$$

where $\|X\|_1 = \text{Tr} \sqrt{X^\dagger X}$ denotes the trace norm (Schatten 1-norm), and ρ_{CQ} takes the following form

$$\rho_{CQ} = \sum_i p_i \Pi_i^A \otimes \rho_i^B, \quad (2)$$

which is a linear combination of the tensor products of Π_i^A (the orthogonal projector in the Hilbert space \mathcal{H}_A) and ρ_i^B (an arbitrary density operator in \mathcal{H}_B), with $\{p_i\}$ being a probability distribution.

For the special case of the two-qubit X states ρ^X whose possible nonzero elements are along only the main diagonal and anti-diagonal [27], the trace distance discord can be derived analytically [28], which is of the following compact form

$$D_T(\rho^X) = \sqrt{\frac{\gamma_1^2 \gamma_{\max}^2 - \gamma_2^2 \gamma_{\min}^2}{\gamma_{\max}^2 - \gamma_{\min}^2 + \gamma_1^2 - \gamma_2^2}}, \quad (3)$$

where $\gamma_{1,2} = 2(|\rho_{23}| \pm |\rho_{14}|)$, $\gamma_3 = 1 - 2(\rho_{22} + \rho_{33})$, $\gamma_{\max}^2 = \max\{\gamma_3^2, \gamma_2^2 + x_{A3}^2\}$, and $\gamma_{\min}^2 = \min\{\gamma_1^2, \gamma_3^2\}$, with $x_{A3} = 2(\rho_{11} + \rho_{22}) - 1$.

If one further consider a specific subset of the X states, i.e., the Bell diagonal states of the form $\rho^{\text{BD}} = \frac{1}{4}[I_2 \otimes I_2 + \vec{c} \cdot (\vec{\sigma} \otimes \vec{\sigma})]$, with $\vec{c} = \{c_1, c_2, c_3\}$ being a three-dimensional vector with elements satisfying $0 \leq |c_i| \leq 1$, and $\vec{\sigma} = \{\sigma_1, \sigma_2, \sigma_3\}$ denotes the standard Pauli matrices, the trace distance discord can be further simplified as [24]

$$D_T(\rho^{\text{BD}}) = \text{int}\{|c_1|, |c_2|, |c_3|\}, \quad (4)$$

which is in fact the intermediate value for the absolute values of the correlation functions c_1 , c_2 , and c_3 .

Different from that of the trace distance discord, the Bures distance geometric discord is defined via the Bures distance $d_B(\rho, \sigma) = 2[1 - \sqrt{F(\rho, \sigma)}]$ between two density operators ρ and σ [25], which is similar with that of the Bures measure of entanglement [29]. Here, we take the definition of Ref. [30], which is of the following form

$$D_B(\rho) = \sqrt{(2 + \sqrt{2})[1 - \sqrt{F_{\max}(\rho)}]}, \quad (5)$$

where $F_{\max}(\rho) = \max_{\chi \in \rho_{CQ}} F(\rho, \chi)$ denotes the maximum of the Uhlmann fidelity $F(\rho, \chi) = [\text{Tr}(\sqrt{\rho \chi \sqrt{\rho}})]^2$. Note that $D_B(\rho)$ in Eq. (5) is normalized, and its square equals to that defined in Ref. [25].

There are several special cases that the evaluation of the Bures distance discord can be simplified. (i) Pure state $|\Psi\rangle$. For this case we have $F_{\max}(|\Psi\rangle) = \mu_{\max}$, with μ_{\max} being the largest Schmidt coefficient of $|\Psi\rangle$ [25]. (ii) The Bell-diagonal states ρ^{BD} , for which we have [30, 31]

$$F_{\max}(\rho^{\text{BD}}) = \frac{1}{2} + \frac{1}{4} \max_{\langle ijk \rangle} \left[\sqrt{(1 + c_i)^2 - (c_j - c_k)^2} + \sqrt{(1 - c_i)^2 - (c_j + c_k)^2} \right], \quad (6)$$

where the maximum is taken over all the cyclic permutations of $\{1, 2, 3\}$. (iii) For the $2 \times n$ -dimensional system, although there is no analytic solution, the maximum of the Uhlmann fidelity can be calculated as [31]

$$F_{\max}(\rho) = \frac{1}{2} \max_{\|\vec{u}=1\|} \left(1 - \text{Tr} \Lambda(\vec{u}) + 2 \sum_{k=1}^{n_B} \lambda_k(\vec{u}) \right), \quad (7)$$

where $\lambda_k(\vec{u})$ represents the eigenvalues of $\Lambda(\vec{u}) = \sqrt{\rho}(\sigma_{\vec{u}} \otimes I_{n_B})\sqrt{\rho}$ in non-increasing order, and $\sigma_{\vec{u}} = \vec{u} \cdot \vec{\sigma}$ with $\vec{u} = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$ being a unit vector in \mathbb{R}^3 , and n_B the dimension of \mathcal{H}_B .

III. THE MODEL

After recalling the basic formalism for the trace distance and the Bures distance geometric discord, we now present the model for our system and the scenario of system-environment coupling. The central system we considered consists of two identical qubits, and they are subject to either of the following two representative structured reservoirs: (i) the independent or (ii) the common zero-temperature reservoir [32]. The corresponding Hamiltonian are given respectively by

$$\hat{H}_i = \omega_0 \sum_n \sigma_+^n \sigma_-^n + \sum_{k,n} (\omega_k^n b_k^{n\dagger} b_k^n + g_k^n b_k^n \sigma_+^n + \text{h.c.}), \quad (8)$$

and

$$\hat{H}_c = \omega_0 \sum_n \sigma_+^n \sigma_-^n + \sum_k \omega_k b_k^\dagger b_k + \sum_{k,n} (g_k b_k \sigma_+^n + \text{h.c.}), \quad (9)$$

where ω_0 and ω_k denote, respectively, the transition frequency of the two qubits and frequency of the reservoir field mode k with the bosonic creation (annihilation) operator b_k^\dagger (b_k) and the system-reservoir coupling constant g_k . Moreover, $\sigma_\pm = (\sigma_1 \pm i\sigma_2)/2$, and the superscript n in the summation runs over the two qubits and their respective reservoir. Note that we considered here that the two qubits are sufficiently separated from each other and, therefore, no direct interactions between them have been taken into account.

The above model was used to study dynamics of entanglement [33–36], entropic discord [37–39], and other related topics [40–42]. The evolution of the two qubits under the system-environment coupling depends on the particular choice of the

spectral density of the reservoir. In this paper, we take the Lorentzian spectral distribution of the following form [32]

$$J(\omega) = \frac{1}{2\pi} \frac{\gamma_0 \lambda^2}{(\omega - \omega_0)^2 + \lambda^2}, \quad (10)$$

where the parameters λ and γ_0 define the spectral width of the reservoir and the decay rate, respectively. They are associated with the reservoir correlation time τ_B and the relaxation time τ_R by $\tau_B \approx \lambda^{-1}$ and $\tau_R \approx \gamma_0^{-1}$, and their relative magnitudes determine the Markovian ($\lambda > 2\gamma_0$) and the non-Markovian ($\lambda < 2\gamma_0$) regimes.

For the above scenario of system-environment coupling, the evaluation of the time-evolved density matrix $\rho(t)$ has already been discussed in the literature [32]. Here, we point out that for the independent reservoir, analytical solutions of $\rho(t)$ can be derived for arbitrary initial states [33], while for the common reservoir, $\rho(t)$ can be solved numerically via the pseudomode approach [35], and for the special case of the initial two-qubit extended Werner-like states, compact form of $\rho(t)$ can also be obtained by using the technique of Laplace transformation [43].

IV. ROBUSTNESS AND PRESERVATION OF THE GEOMETRIC DISCORD

With the help of the above preliminaries, we now begin our discussion about robustness of the geometric discord $D_T(\rho)$ and $D_B(\rho)$ under the Lorentzian structured reservoir. We will take $|\Phi\rangle = \alpha|10\rangle + \sqrt{1 - \alpha^2}|01\rangle$ as the initial state of the two qubits, and for the sake of simplicity, we will do not list the explicit form of $\rho(t)$ here as they can be easily written via the methods mentioned above [33, 43].

When considering the trace distance discord $D_T(\rho)$ for the initial state $|\Phi\rangle$, our calculation shows that it is a symmetric quantity with respect to $\alpha^2 = 0.5$. In Fig. 1, we plotted dynamics of $D_T(\rho)$ versus the scaled time $\gamma_0 t$ for the case of the two qubits subject to independent reservoirs, from which one can note that $D_T(\rho)$ takes its maximum at $\alpha^2 = 0.5$, and decreases with the increase of $|\alpha^2 - 0.5|$. For fixed α^2 , $D_T(\rho)$ decays monotonically with increasing $\gamma_0 t$ in the Markovian regime [Fig. 1(a)], while it exhibits damped oscillations in the non-Markovian regime [Fig. 1(b)], and suffers instantaneous disappearance at the critical time $t_n = 2[n\pi - \arctan(d/\lambda)]/d$, with $d = \sqrt{2\gamma_0\lambda - \lambda^2}$ and $n \in \mathbb{Z}$ [33]. As there are no direct interactions between the two qubits, and the two qubits interact respectively with their own independent reservoir, the revivals of $D_T(\rho)$ after its instantaneous disappearance in the non-Markovian regime is induced by the memory effects of the reservoir.

When the two qubits are subject to the common reservoir, we plotted in Fig. 2 dynamics of $D_T(\rho)$ versus $\gamma_0 t$ for the initial state $|\Phi\rangle$. Different from that of the independent reservoirs, $D_T(\rho)$ here does not behave as monotonic functions of $|\alpha^2 - 0.5|$. In the Markovian regime as shown in Fig. 2(a), $D_T(\rho)$ decays asymptotically to zero for $\alpha^2 = 0.5$, and for other α^2 , they first decay to the minimum 0, and then turn out

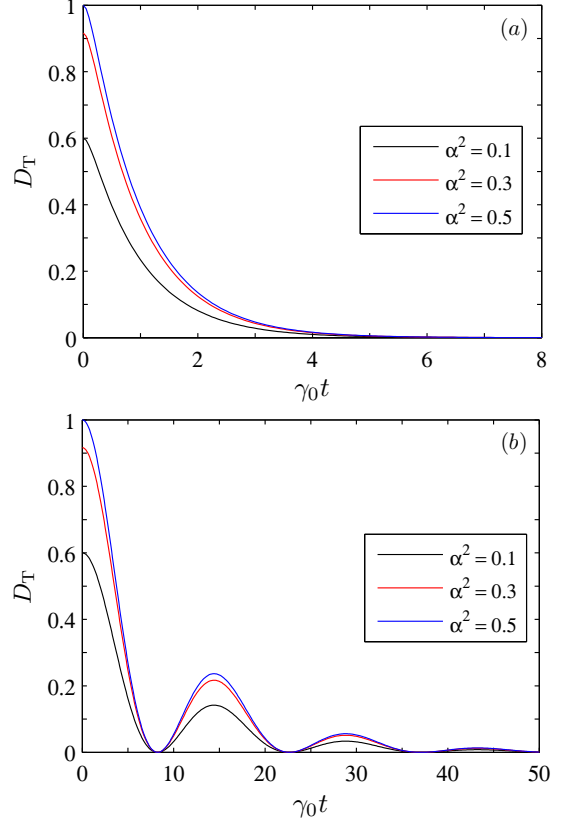


FIG. 1: (Color online) Trace distance discord $D_T(\rho)$ versus $\gamma_0 t$ for the initial states $|\Phi\rangle$ in independent reservoirs, where the parameter λ is chosen to be $\lambda = 10\gamma_0$ (a) and $\lambda = 0.1\gamma_0$ (b), respectively.

to be increased to certain steady-state values in the infinite-time limit. In the non-Markovian regime, as can be seen from Fig. 2(b), $D_T(\rho)$ oscillates with damped amplitudes for $\alpha^2 = 0.5$, and exhibits very complicated behaviors for other values of α^2 , which are induced by the combined effects of the non-Markovianity and the reservoir-mediated interaction between the two qubits. But in the long-time limit, the reservoir-mediated interaction between the two qubits dominates, and $D_T(\rho)$ arrives at steady-state values which are completely the same as those for the Markovian case in Fig. 2(a).

We now turn to discuss robustness of the Bures distance discord under the system-environment coupling. Due to the different distance measures adopted, one may expect that $D_B(\rho)$ and $D_T(\rho)$ will exhibit different behaviors. In Fig. 3, we showed plots of $D_B(\rho)$ versus $\gamma_0 t$ for the initial state $|\Phi\rangle$ in the non-Markovian regime. First, one can note that $D_B(\rho)$ is no longer a symmetric quantity with respect to $\alpha^2 = 0.5$, and this is a difference between $D_B(\rho)$ and $D_T(\rho)$. But after a critical point $\gamma_0 t_c$ which is determined by α^2 and the system-environment coupling parameters, the curves for $D_B(\rho)$ with α^2 and $|\alpha^2 - 1|$ converge. Moreover, one can see that for the case of independent reservoirs, $D_B(\rho)$ behaves as damped oscillations when $t > t_c$, and disappears instantaneously at the same critical time t_n as that for $D_T(\rho)$.

For the case of common reservoir, as can be seen from Fig.

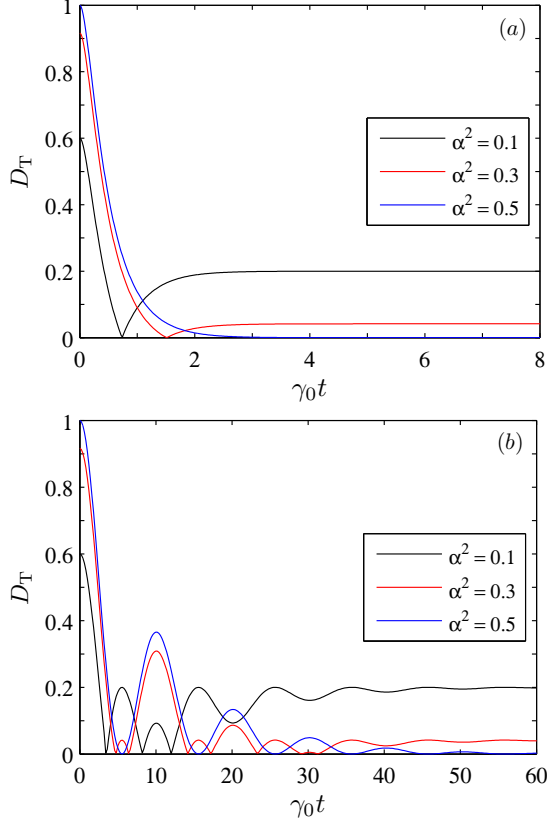


FIG. 2: (Color online) Trace distance discord $D_T(\rho)$ versus $\gamma_0 t$ for the initial states $|\Phi\rangle$ in common reservoir, where the parameter λ is chosen to be $\lambda = 10\gamma_0$ (a) and $\lambda = 0.1\gamma_0$ (b), respectively.

3(b), the memory effects of the reservoir and the reservoir-mediated interaction between the considering qubits together induce more complicated behaviors of $D_B(\rho)$ than that for the independent reservoirs. But beyond the short $\gamma_0 t$ region, $D_B(\rho)$ oscillates with fixed periods which are independent of the values of α^2 . Particularly, one can note that apart from the special case of $\alpha^2 = 0.5$ for which $D_B(\rho)$ behaves as damped oscillations and disappears when $\gamma_0 t \rightarrow \infty$, the peak values $D_B^{\text{peak}}(\rho)$ for the initial states $|\Phi\rangle$ with other α^2 remain unchanged during their time evolution process, and these peak values equal to their steady-state values $D_B^{\text{stead}}(\rho)$ in the infinite-time limit, at which the indirect interaction between the two qubits induced by their simultaneous interactions with the common reservoir dominates.

Another phenomenon needs to be pay attention to is that for the cases of $\alpha^2 = 0.1$ and 0.9 as represented by the black and cyan curves in Fig. 3(b), the steady-state values $D_B^{\text{stead}}(\rho)$ are nearly the same as those for the initial states $|\Phi\rangle$. Meanwhile, as these steady-state values are increased with increasing values of $|\alpha^2 - 0.5|$, one thus expects naturally that for very small or very large α^2 , the Bures distance discord may be enhanced and turns out to be larger than its initial value. This is indeed the case not only for the Bures distance discord, but also for the trace distance discord.

As exemplified plots, we illustrated in Fig. 4 the $\gamma_0 t$ depen-

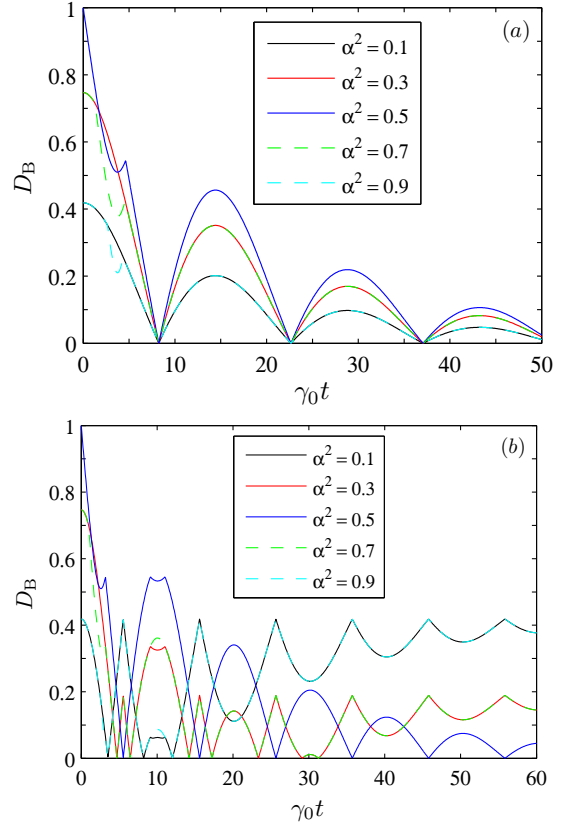


FIG. 3: (Color online) Bures distance discord $D_B(\rho)$ versus $\gamma_0 t$ for the initial states $|\Phi\rangle$ in independent reservoirs (a) and common reservoir (b), both with $\lambda = 0.1\gamma_0$.

dence of $D_T(\rho)$ and $D_B(\rho)$ with various values of $\alpha^2 < 0.1$. When considering the trace distance discord, our numerical results show that if $\alpha^2 \lesssim 0.0286$ [e.g., the blue curve for $\alpha^2 = 0.02$ in Fig. 4(a)], the steady-state value of $D_T(\rho)$ becomes larger than its initial value, and for the special case of $\alpha^2 = 0$ which corresponds to the classical state $|\Phi\rangle = |10\rangle$, the noisy effects of the common reservoir can even generate trace distance discord, with its maximum be of about 0.5. We point out here that the experimental generation of QD for two ionic qubits via noisy processes has been reported in a very recent work [44], while for generating QD by local operations, a general approach and powerful result is in Ref. [45], where it is proved that any separable but quantum correlated states can be generated from classical states in higher dimensions via local tracing.

For the Bures distance discord, as can be seen from Fig. 4(b), the peak values $D_B^{\text{peak}}(\rho)$ for $\alpha^2 \neq 0$ equal to their steady-state values $D_B^{\text{stead}}(\rho)$ and are larger than their initial values. When $\alpha^2 = 0$, as illustrated by the green dash-dotted curve in Fig. 4(b) which corresponds to vanishing $D_B(\rho)$ at the initial time, the noisy process of the common reservoir can also generate Bures distance discord. For the initial state $|\Phi\rangle = |10\rangle$, the maximum of $D_B(\rho)$ is of about 0.588, while its steady-state value is of about 0.495, and these are achieved within the scaled time interval $\gamma_0 t \in [0, 1000]$. These phe-

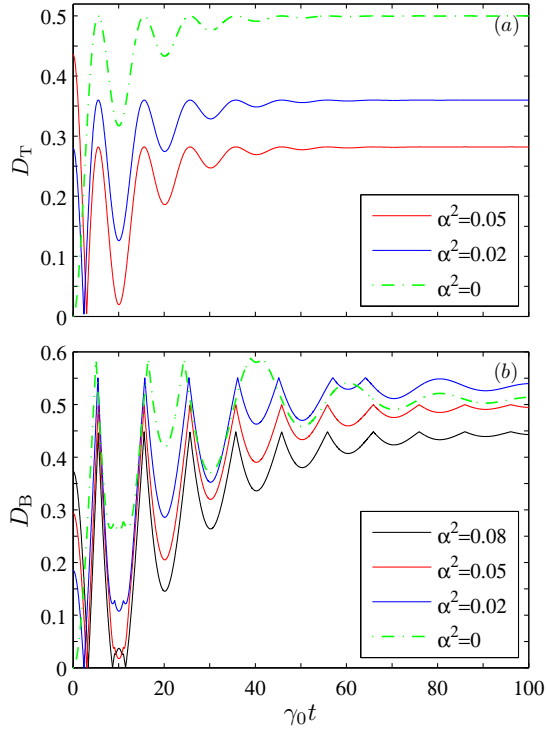


FIG. 4: (Color online) Trace distance discord $D_T(\rho)$ and Bures distance discord $D_B(\rho)$ versus $\gamma_0 t$ for the initial states $|\Phi\rangle$ in common reservoir with small values of α^2 and $\lambda = 0.1\gamma_0$.

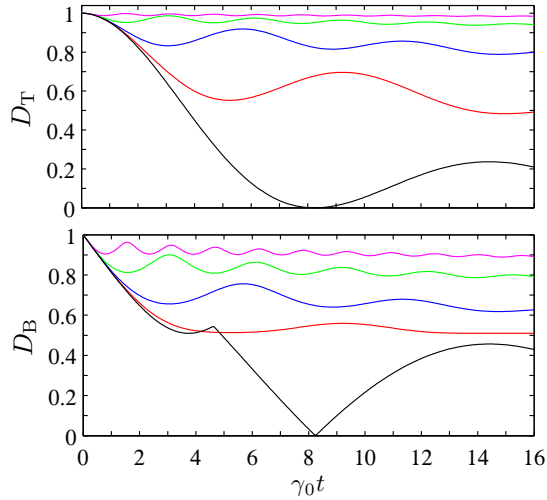


FIG. 5: (Color online) Preservation of $D_T(\rho)$ and $D_B(\rho)$ in the non-Markovian regime of the independent reservoirs. The initial state is $|\Phi\rangle$ with the parameters $\alpha^2 = 0.5$, $\lambda = 0.1\gamma_0$, and the curves from bottom to top correspond to $\delta/\gamma_0 = 0, 0.5, 1, 2$, and 4 .

nomena show that for certain family of the initial states, the distance measures of geometric discord can be preserved well, or even be improved and generated, and it might indicate a robust pathway to quantum protocols based on them.

After making clear their dynamical behaviors, we now try to provide possible methods for preserving the trace distance

and the Bures distance discord. Of course, for certain family of the initial states, they can be preserved well or even be enhanced by the common reservoir, and thus no other specific manipulations are needed for our purpose. Here, we will further show that for the case of independent reservoirs, the geometric discord of both $D_T(\rho)$ and $D_B(\rho)$ can also be preserved well by the following strategy.

To be explicitly, we introduce a detuning to the transition frequency ω_0 by an amount δ , i.e., we replace the parameter ω_0 in Eq. (10) with the central frequency $\omega_c = \omega_0 - \delta$, and show that it can serve as an efficient parameter for tuning both $D_T(\rho)$ and $D_B(\rho)$. To this end, we displayed in Fig. 5 two exemplified plots for $D_T(\rho)$ and $D_B(\rho)$ versus $\gamma_0 t$, from which one can note that both of them can be enhanced by introducing detuning, and for the chosen parameters in Fig. 5, they begin to oscillate weakly around their initial values when $\delta = 4\gamma_0$. Therefore, it is reasonable to conjecture that for the case of very large detuning, both $D_T(\rho)$ and $D_B(\rho)$ will maintain their initial values during the time evolution process, and the two kinds of geometric discord are thus frozen. Such frozen discord provides useful resource for future operations for quantum protocols relied on them.

V. SUMMARY

In summary, we have investigated robustness of the trace distance and the Bures distance geometric discord against decoherence. By subjecting the considered qubits to structured reservoirs with the Lorentzian spectral density, we showed that for certain family of the initial states, the two well-defined geometric discord can be preserved or improved in the common reservoir. Particularly, the noisy process induced by the common reservoir can even generate geometric discord from the classical states. Moreover, we showed that by introducing detuning to the transition frequency ω_0 of the qubits, an efficient monitoring and long-time preservation of the geometric discord in non-Markovian independent reservoirs is also possible, and this inherent robustness might indicate a pathway to quantum protocols for the open quantum system.

While these two distance measures of geometric discord are well defined and have important conceptual implications [24–26], they may exhibit remarkable features, such as the long-time preservation, improvements, and generation in the noisy environments as revealed in this paper, and these make them also important for certain quantum tasks, which represents a significant challenge and remains as a direction for future research.

ACKNOWLEDGMENTS

This work was supported by NSFC (11205121), NSF of Shaanxi Province (2010JM1011), and the Scientific Research Program of the Education Department of Shaanxi Provincial Government (12JK0986).

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